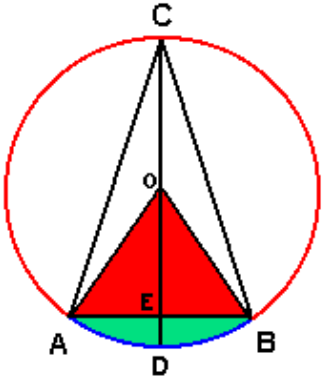


## Circle Formulas

[www.1728.org/circpart.htm](http://www.1728.org/circpart.htm)



Lines AO, OD and OB are called radii.  
 Lines AC, BC and AB are called chords.  
 The angle ACB is an inscribed angle.  
 The angle AOB is a central angle.  
 The curved line AB is called an arc.  
 Line OE is the apothem and is the height of triangle AOB.  
 Line ED is the segment height or the sagitta - a rarely used term.  
 The length of an arc equals  $(\text{Central Angle} / 180^\circ) \cdot \pi \cdot r$   
 The green area is called a segment.  
 The red triangle plus the green segment is called a sector.  
Sector Area =  $(\text{Angle AOB}^\circ / 360^\circ) \cdot \pi \cdot r^2$   
Sector Area =  $(\text{Angle AOB} \div 2) \cdot r^2$  (for radians)  
Segment Area = Sector Area Minus Triangle AOB Area.

[www.1728.com](http://www.1728.com)

### 1) Radius and Central Angle

We know the central angle is AOB and so angle AOE =  $\frac{1}{2}$  central angle

From trigonometry we know that the sine of angle AOE =  $AE/AO$

So, line  $AE = \text{sine of angle AOE} \cdot \text{line AO}$

Using the Pythagorean Theorem line  $OE^2 = AO^2 - AE^2$

Segment Height ED = Radius AO - Apothem OE

### 2) Radius AO & Chord AB

$AE = \frac{1}{2}AB$

From the Pythagorean Theorem  $OE^2 = AO^2 - AE^2$

Segment Height ED = Radius AO - Apothem OE

Angle AOE = arc tangent (AE/OE)

Central Angle AOB =  $2 \cdot \text{Angle AOE}$

### 3) Radius AO & Segment Height ED

Apothem OE = Radius AO - Segment Height ED

Angle AOE = arc tangent (AE/OE)

Central Angle AOB =  $2 \cdot \text{Angle AOE}$

### 4) Radius AO & Apothem OE

Segment Height ED = Radius AO - Apothem OE

Angle AOE = arc tangent (AE/OE)

Central Angle AOB =  $2 \cdot \text{Angle AOE}$

5) Radius AO & Arc AB

$$\text{Circumference} = 2 \cdot \pi \cdot \text{radius AO}$$

$$\text{Central Angle AOB} = (\text{Arc AB} / \text{Circumference}) \cdot 360$$

$$\text{Angle AOE} = \text{Central Angle AOB} / 2$$

$$\text{Chord AB} = 2 \cdot \text{sine}(\text{Angle AOE}) \cdot \text{radius}$$

6) Chord AB & Segment Height ED

This is where the "intersecting chord theorem" *really* comes in handy.

$$CE \cdot ED = AE \cdot EB$$

$$CE = (AE \cdot EB) / ED$$

Since  $AE = EB = \frac{1}{2}AB$  then:

$$CE = (\frac{1}{2}AB \cdot \frac{1}{2}AB) / ED$$

$$CE = AB^2 / 4 \cdot ED$$

$$\text{Radius AO} = (CE + ED) / 2$$

$$\text{Apothem OE} = \text{Radius AO} - \text{Segment Height ED}$$

$$\text{Angle AOE} = \text{arc tangent}(AE/OE)$$

$$\text{Central Angle AOB} = 2 \cdot \text{Angle AOE}$$

7) Chord AB & Apothem OE

$$AE = \frac{1}{2}AB$$

From the Pythagorean Theorem

$$\text{Radius AO}^2 = \text{OE}^2 + \text{AE}^2$$

$$\text{Segment ED} = \text{Radius AO} - \text{Apothem OE}$$

$$\text{Angle AOE} = \text{arc tangent}(AE/OE)$$

$$\text{Central Angle AOB} = 2 \cdot \text{Angle AOE}$$

8) Segment Height ED & Apothem OE

$$\text{Radius AO} = \text{Segment Height ED} + \text{Apothem OE}$$

$$\text{Angle AOE} = \text{arc tangent}(AE/OE)$$

$$\text{Central Angle AOB} = 2 \cdot \text{Angle AOE}$$

From the Pythagorean Theorem

$$AE^2 = \text{AO}^2 - \text{OE}^2$$

$$\text{Chord AB} = 2 \cdot AE$$

9) Chord AB & Arc Length AB (curved blue line)

There is *no formula* that can solve for the other parts of a circle if you only know the chord and the arc length.

There is a procedure called Newton's Method which can produce an answer. To try it, click the link [here](#) and then scroll about  $\frac{3}{4}$  of the way down to "A Real World Example" where we have a worked out example.